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A Glimpse of Rectangles in Connection with Gopa numbers of First Kind

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Abstract:

This paper has two sections I and II. **Section I** exhibits rectangles, where, in each rectangle, twice the area added with its semi-perimeter is represented by a Gopa number of first kind. **Section II** exhibits rectangles, where, in each rectangle, twice the area minus its semi-perimeter is represented by a Gopa number of first kind. The total number of primitive and non-primitive rectangles is also given.

Keywords:

Rectangles, Gopa number of first kind, Primitive rectangles,
Non-Primitive rectangles.

Introduction:

The diophantine problems connecting geometrical representations with special patterns of numbers are presented in [1-21]. This paper concerns with the problem of finding rectangles such that, in each rectangle, twice the area added with its semi-perimeter as well as twice the area minus its semi-perimeter is represented by a Gopa number of first kind. The total number of primitive and non- primitive is also given.

It seems that the above problems have not been considered earlier.

. Definition: Gopa numbers of the First kind

Let N be a non-zero positive integersuch that$N = P \times Q$, where P and Q are distinct primes.

If the relation

Sum of the divisors of N = Product of the sum of the divisors of P, Q

= a perfect square

then, the integer N is referred as Gopa number of the first kind

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Examples: 22,94,119,214,217,382,497,517,527,679,1177,2101,5029

Method of Analysis:

Let R be a rectangle with dimensions x and y. Let A and S represent the Area and Semi-perimeter of R. If g .c. d . (x,y) =1,then R is referred as primitive rectangle. Otherwise, i is called non-primitive rectangle.

Section-I : $2A+S = Gopa\ number\ of\ the\ first\ kind$

The problem under consideration is mathematically equivalent to solving the binary quadratic diophantine equation represented by

$$2xy+(x+y)=\alpha \tag{I.1}$$

where α is a Gopa number of the first kind.

Rewrite (I. 1) as

$$x=\frac{\alpha - y}{2y+1} \tag{I.2}$$

Given α, it is possible to find x in integers for suitable y in integers. The following Table 1.1 exhibits the Gopa number of the first kind with their corresponding rectangles satisfying (I.1):

Table 1.1: $2A+S=\alpha$

Gopa number of the first kind (α)	$R(x,y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
22	(1,7), (7,1),(2,4),(4,2)	-	4
94	(1,31),(3,13),(4,10),(10,4),(13,3),(31,1)	-	6
214	(1,71), (5,19),(6,16),(16,6),(19,5),(71,1)	-	6
217	(1,72), (2,43), (7,14), (14,7),(43,2),(72,1)	6	-
382	(1,127), (2,76),(4,42),(7,25),(8,22),(22,8), (25,7),(42,4),(76,2),(127,1)	-	10

.

Section- II : $2A-S =$ **Gopa number of the first kind**

The problem under consideration is mathematically equivalent to solving the binary quadratic diophantine equation represented by

$$2xy-(x+y)=\alpha \qquad \text{(II.1)}$$

where α is a Gopa-Vidh number.
Rewrite (II.1) as

$$x=\frac{\alpha+y}{2y-1} \qquad \text{(II.2)}$$

Given α, it is possible to find x in integers for suitable y in integers. The following Table 2.1 exhibits the Gopa number of the first kind with their corresponding rectangles satisfying (II.1)

Table 2.1: $2A-S=\alpha$

Gopa number of the first kind (α)	$R(x,y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
22	(1,23), (3,5),(5,3),(23,1)	-	4
94	(1,95), (2,32),(4,14),(5,11),(11,5),(14,4),(32,2),(95,1)	-	8
214	(1,215), (2,72), (6,20), (7,17),(17,7),(20,6),(72,2),(215,1)	-	8
217	(1,218), (2,73),(3,44),(8,15),(15,8),(44,3),(73,2),(218,1)	8	-
382	(1,383), (2,128), (3,77), (5,43),(8,26),(9,23),(23,9),(26,8),(43,5),(77,3),(128,2),(383,1)	-	12

.

Conclusion:

In this paper, we have presented rectangles such that, in each rectangle,twice the area added with its semi-perimeter as well as twice the area minus the semi-perimeter is represented by a Gopa number of the first kind.

To conclude, one may search for rectangles with other characterization in connection with higher order .Gopa numbers of the first kind.

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